Single-Frequency Relative Q Measurements Using Perturbation Theory

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Abstract—**Traditionally, Q measurement requires a nonzero frequency bandwidth or time period. Contrasted to this, the principle of a new single-frequency relative Q measurement method is developed. It is found that Q is directly or inversely proportional to the normalized input resonant resistance if a moderate perturbation condition is satisfied. Theoretical proof and experimental verification of the single-frequency method's validity are presented. Consequently, a relative Q, often used in dielectric measurements, can be measured using a much simpler measurement system. Moreover, error analysis shows that, in making a relative Q measurement, the error in the single-frequency method is smaller than that in the traditional bandpass method when using a reflectometer.**

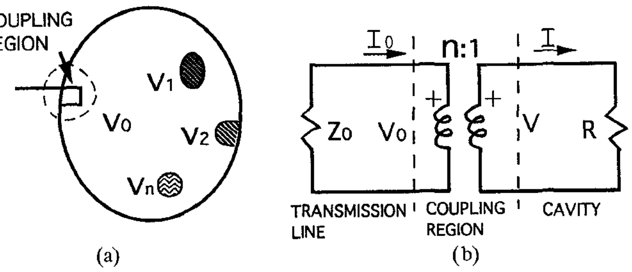

Fig. 1. (a) Resonant cavity containing disturbances $V_1, V_2 \ldots V_n$, and the coupling device. (b) The cavity equivalent circuit at resonance. V_0 is the empty cavity volume. $V_1, V_2 \ldots V_n$ are relatively far from the coupling region.

I. Introduction

Q-factor determination is one of the basic microwave techniques for the characterization of a resonator and extends to the determination of electromagnetic and nonelectromagnetic properties of a material such as dielectric constant, loss factor, moisture content and density.

Over the past fifty years, a great variety of Q measurement methods were developed, but all fall into essentially two categories, namely, the bandpass method and the time decrement method [1]. The former is based on a resonator's response to CW signals in the vicinity of its resonant frequency [1]. The latter involves observing the resonator's transient response to the sudden application or removal of an exciting signal at or near the resonant frequency. Consequently, these methods require either the excitation of a measurement signal over a frequency band or over a given time period, making it impossible to measure Q at a single frequency. Therefore, when the Q-factor of a single-frequency system needs to be measured, a sweep oscillator must be added to the system (see, for example, the method used in high-temperature microwave dielectric measurements [2]), thus increasing system complexity and cost and causing interference with the original system frequency and system operation.

In this paper, based on the perturbation theory, we prove the possibility of measuring Q at single frequency, which only requires an input resistance measurement of a cavity at its resonant frequency, while a moderate perturbation condition is satisfied. Applying this principle, a new method for a single-frequency relative Q measurement has been developed. Experimental verification of the method and an error analysis are also presented.

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II. Theoretical Basis

A. Input Resonant Resistance and Q

It is well known that the resonant resistance R on the cavity side of the equivalent coupling plane (Fig. 1) is defined, for magnetic coupling, as

$$R = \frac{2P}{\left(\oint \overline{H} \cdot d\overline{L}\right)^2} = \frac{2P}{I^2} \tag{1}$$

and for electric coupling, as

$$R = \frac{\left(\int_a^b \overline{E} \cdot d\overline{L}\right)^2}{2P} = \frac{V^2}{2P} \tag{2}$$

where P is the power dissipated in the cavity, and I and V are the equivalent current and the equivalent voltage on the cavity side of the coupling device respectively. The integration path for H encloses the path where the current I is defined. Similarly, the integration limits, a and b, are the points between which the equivalent voltage V is defined. We need to analyze only one of the two coupling cases since the results of the other can be obtained by the principle of duality. Here we choose to analyze the magnetic coupling case.

By definition, the Q factor is

$$Q \equiv \frac{\omega W}{P} \tag{3}$$

where ω is the angular resonant frequency and W is the energy stored in the cavity. Substituting for P from (3) into (1), the resistance at resonance is represented by

$$R = \frac{2\omega W}{\left(\oint_c \overline{H} \cdot d\overline{L}\right)^2 Q} \tag{4}$$

Transforming through the coupling device and normalizing with respect to the characteristic impedance of the input transmission line, Z_0, we obtain the normalized input resistance $\overline{R}$ looking into the cavity from the outside as

$$\overline{R} = \frac{n^2 R}{Z_0} = \frac{2n^2 \omega S}{Z_0 Q} \tag{5}$$

where n is the current transformer ratio of the coupling mechanism and S is defined as the ratio of the stored energy in the cavity to the square of the integrated magnetic field around the coupling device, i.e., the square of the RF current in the coupling device of the cavity, so that

$$S = \frac{W}{\left(\oint_c \overline{H} * d\overline{L}\right)^2} \tag{6}$$

Alternatively, Q can also be written as a function of $\overline{R}$

$$Q = \frac{2n^2 \omega S}{Z_0 \overline{R}} \tag{7}$$

Up to this point, no approximations have been made. Therefore, an absolute value of Q can be obtained by measuring the input resistance $\overline{R}$ under the resonant condition provided that Z_0, n, ω, and S are known. However, this is realistic only for some well-defined resonator geometries such as the circularly cylindrical cavity, the rectangular cavity, and the coaxial cavity, with coupling mechanisms that can be fully analyzed to obtain the values for n and S.

The real significance of (7), however, is the fact that it reveals how a Q can be measured without resorting to a band of frequency or a given time increment. What may, practically, be even more significant is that a relative value of Q can be obtained with sufficient accuracy by simply measuring $\overline{R}$ even without solving for n, S, and ω if a moderate perturbation condition is satisfied, as we will proceed to prove next. Consequently, the relative Q measurement can be used in practical single-frequency dielectrometers.

B. Perturbation Effect

In most microwave dielectric measurements, the variations of the parameters of the dielectric, causing the perturbation, are well within the range allowed by perturbation theory. Therefore, the analysis of the relationship between Q and $\overline{R}$ under a perturbation condition is significant in this sense.

Equation (7) shows that, besides $\overline{R}$, the four quantities Z_0, n, ω, and S would affect Q. Since Z_0 is a known constant we only have to examine how n, ω, and S vary when a resonator is perturbed.

For a given field configuration in the coupling region, the value of the transformer ratio n is decided by the geometric structure of a coupling mechanism. In other words, n will not vary as long as the field configuration in the neighborhood of the coupling mechanism remains unchanged. Notice that this is a weaker condition than demanded in the conventional perturbation theory, where invariance of the fields throughout the whole cavity is required except in the locally perturbed region. If the coupling mechanism is situated far enough from the disturbed field region, as is usual in practice, the above perturbation condition is satisfied quite rigorously. Moreover, when the dominant fields at the coupling port are different from those in the perturbed region, the effect of the perturbation on the coupled field will be still further reduced [3] and hence can be assumed negligible. From this we conclude that n would remain a constant as long as a moderate perturbation condition is satisfied.

Consider now how the perturbation affects S. Perturbation theory requires that, on introducing disturbances, the fields may change considerably in magnitude but the field configurations or the patterns will not change appreciably except in the region of the disturbances. Say that, as a result of the disturbances, the existing field magnitude changes from H_0 to

$$H = hH_0 \tag{8}$$

where h is a simple scale factor. By (6) and (8), and noting that the energy of $W = (1/2)\mu \int H^2\, dv$, we obtain

$$S = \frac{\mu h^2 \int v \overline{H}_0^2\, dV}{2h^2 \left(\oint_c \overline{H}_0 d\overline{L}\right)^2} = S_0 \tag{9}$$

where S_0 is the S for the undisturbed resonator, and $V = V_0$. It is evident that the parameter S is invariant under the perturbation assumption. Now Q can be expressed as

$$Q = \frac{2}{Z_0} n_0^2 S_0 \omega_0 \left(1 + \frac{\Delta\omega}{\omega_0}\right) \frac{1}{\overline{R}} = \frac{C_0}{\overline{R}} \left(1 + \frac{\Delta\omega}{\omega_0}\right) \tag{10}$$

where $C_0 = 2n_0^2 S_0 \omega_0 / Z_0$ is a constant for a resonator satisfying the perturbation assumptions. The subscript 0 refers to the quantities before perturbation.

In order to examine the perturbation effect on ω, consider that the field is disturbed and redistributed, thereby altering the resonant frequency. The variation of the resonant frequency can be represented by the well-known cavity perturbation formula

$$\frac{\Delta\omega}{\omega_0} = \frac{\int_{V_0} \left[\left(\overline{E}_1 * \overline{D}_0 - \overline{E}_0 * \overline{D}_1\right) + \left(\overline{H}_1 * \overline{B}_0 - \overline{H}_0 * \overline{B}_1\right)\right] dV}{\int_{v_0} \left[\overline{E}_0 * \left(\overline{D}_0 + \overline{D}_1\right) - \overline{H}_0 * \left(\overline{B}_0 + \overline{B}_1\right)\right] dV} \tag{11}$$

where $\overline{E}_1$, $\overline{D}_1$, $\overline{H}_1$ and $\overline{B}_1$ are the small incremental changes in $\overline{E}_0$, $\overline{D}_0$, and $\overline{H}_0$ and $\overline{B}_0$ due to the perturbation. The results for many specific cases can be found in the literature [4]. Nevertheless, it is this frequency shift that is the essential quantity in the standard perturbation theory and in its applications. Yet, from the point of view of the Q relation in (10), the effect of the frequency shift is negligible because, in any valid perturbation case, the frequency shift $\Delta\omega/\omega$ is much smaller than unity. For example, 2% of $\Delta\omega/\omega$ may be considered to be quite a large perturbation for a microwave cavity, but it contributes a Q error of only 2% according to (10). For this reason, neglect of $\Delta\omega/\omega$ has little impact on the Q value. Consequently, the Q is very simply related to the normalized input resistance of a resonating cavity by

$$Q = \frac{C_0}{\overline{R}} \tag{12}$$

For the electric coupling case, by the principle of duality, we obtain

$$Q = C_e \overline{R} \tag{13}$$

where C_e is a new constant.

It should be pointed out that our use of an ideal transformer model representing the coupling device in no way limits the generality of the above conclusions.

Note that $\overline{R}$ is related to the absolute reflection coefficient $|\Gamma|$ by

$$\overline{R} = \frac{1 \pm |\Gamma|}{1 \pm |\Gamma|} \tag{14}$$

where the choice of the sign depends on whether the cavity is overcoupled or undercoupled, respectively. Applying (12)–(14), we can determine the relative Q-factor through the measurement of just the reflection coefficient. Only if an absolute value of Q needs to be found must a value for C_0 be determined.

C. Detuning and Retuning

In perturbation applications, two cases are very common. One is the detuning case, where a dielectric disturbance, V_1 as in Fig. 1, causes a resonant frequency shift. The other is the retuning case, where, after a first disturbance, the resonant frequency is retuned via a second disturbance, say V_2, to the original unperturbed resonator frequency. Therefore, the retuning results in true single frequency operation since a fixed frequency has been maintained. To show that the relation between Q and $\overline{R}$ still holds for the retuning case, we consider the following argument.

For the detuning case, the frequency shift can be derived from (11) as

$$\left.\frac{\Delta\omega}{\omega_0}\right|_{V_1} = \frac{\int_{V_1}\left(\overline{E}_1 * \overline{D}_0^* - \overline{E}_0^* * \overline{D}_1\right) dV}{4W_0} \tag{15}$$

where

$$W_0 = \frac{1}{4}\int_V \left(\overline{E}_0^* * \overline{D}_0 + \overline{H}_0^* * \overline{B}_0\right) dV$$

is the energy in the unperturbed cavity.

For the retuning case, suppose we slightly change the volume of the cavity by an appropriate second disturbance of volume V_2 to pull the disturbed resonant frequency back to its undisturbed value. This is usually done by adjusting a short circuit plunger or a tuning stub. However, a dielectric property or magnetic permeability variation is also allowed as the secondary disturbance in lieu of a physical volume variation. According to perturbation theory, the frequency shift due to a volume variation is given by [4]

$$\left.\frac{\Delta\omega}{\omega_0}\right|_{V_2} = \frac{\int_{V_2}\left(\overline{H}_0^* * \overline{B}_0 - \overline{E}_0^* * \overline{D}_0\right)}{4W_0} \tag{16}$$

Maintaining the resonant frequency constant requires

$$\left.\frac{\Delta\omega}{\omega_0}\right|_{V_1} = -\left.\frac{\Delta\omega}{\omega_0}\right|_{V_2} \tag{17}$$

Substituting (15) and (16) into (17), and assuming W_0 constant, we obtain

$$\int_{V_1}\left(\overline{E}_1 * \overline{D}_0^* - \overline{E}_0^* * \overline{D}_1\right) dV + \int_{V_2}\left(\overline{H}_0^* * \overline{B}_0 - \overline{E}_0^* * \overline{D}_0\right) dV = 0 \tag{18}$$

which shows that, to maintain a fixed resonant frequency, the addition of the energy variations caused by the two disturbances must be zero. In other words, the energy in the twice-disturbed cavity is equal to that of the undisturbed cavity. Therefore, not only does the frequency remain unchanged but so does the factor S since the energy W remains invariant, as does the coupling loop current given by $\oint_c \overline{H} * d\overline{L}$, on the assumption that the coupling region is far away from the disturbances. Consequently, the Q measured in the retuned case will be more accurate than that measured in the detuned case.

In summary, it is found that, depending on whether electric or magnetic coupling is used, Q is directly or inversely proportional to $\overline{R}$ once the perturbation conditions are satisfied. Therefore, the relative Q can be obtained from a reflection measurement at a single frequency. To achieve this, the critical stipulation is that the field configuration in the coupling region remain undisturbed even though the field amplitudes change. In addition, maintaining a fixed frequency by introducing a secondary disturbance further improves the accuracy.

III. Experimental Verification

In this section, the conclusions based on the theoretical analysis in the previous section are verified experimentally. In order to ensure greater credibility, not one but two cavities, different in structure and Q range, have been employed in the experiment for the verification. One is a circularly cylindrical TM_{013} mode cavity and the other is a re-entrant coaxial cavity. The experiments were conducted in S-band. Comparison data for the Q factor were obtained from the traditional bandpass method.

A typical swept-frequency reflectometer is used for the bandpass Q measurement, as shown in Fig. 2. Using this method, we can obtain the cavity Q, Q_T, by

$$Q_T = (1 + \beta) Q_L \tag{19}$$

where β is the coupling coefficient. The loaded Q, Q_L, is given by

$$Q_L = k\frac{f_0}{\Delta f} \tag{20}$$

where k depends on the power level at which Δf is measured. For example, if the 3-dB power level is chosen, $k = 1$.

On the other hand, the cavity Q obtained through the single-frequency method is designated as Q_p, and is given by (12), i.e.,

$$Q_P = \frac{C_0}{\overline{R}} \tag{21}$$

with the constant C_0 determined by an independent measurement using the bandpass method mentioned. $\overline{R}$ is obtained

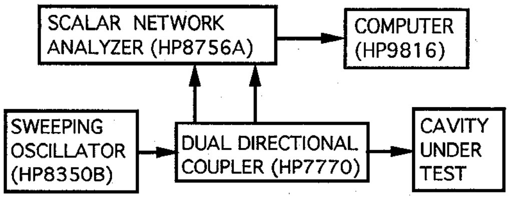

Fig. 2. Experimental reflectometer.

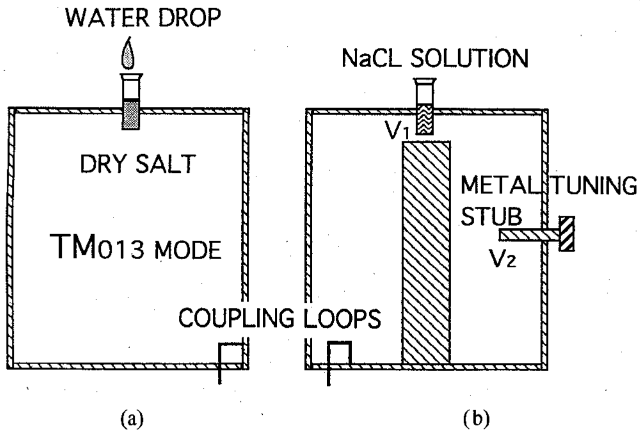

Fig. 3. Test cavity arrangements. a) Circularly cylindrical cavity. A small drop of water sliding into dry salt greatly increases the loss factor resulting in a significant Q drop, while the resonant frequency remains nearly constant. b) Re-entrant coaxial cavity. Insertion of 0.1–N NaCl solution (V_1) into the cavity in the high E–field gap region changes both Q and resonant frequency. The metal tuning stub (V_2) is used to pull the resonant frequency back to the unperturbed value.

through (13) by an absolute reflection coefficient measurement at the resonant frequency. Novel software for the HP8756 scalar network analyzer [2] collected the data of the pertinent resonant curve including Δf, f_0 and $|\Gamma|$, which simultaneously contain the information for both the bandpass and the single-frequency method, ensuring identical electromagnetic conditions for both tests. Therefore, any source frequency and power instability will not affect the relative comparison between these two different methods, making the comparison more direct and reliable.

A. Cavity Description

The geometrical structures of the two cavities are shown in Fig. 3. Two things about the construction of the cavities should be mentioned. First, they both have a hole for introducing the disturbances, mostly lossy materials, by which the Q variation can be realized without changing cavity geometry. Second, the coupling mechanisms are located far from the disturbing sources and both are loop-coupled to the H-field to achieve as high an isolation as possible between the coupling mechanisms and the disturbances placed in a maximum E-field. Two different procedures of introducing the disturbances were used for the cylindrical cavity and the re-entrant coaxial cavity. The first was used to confirm the validity of the single-frequency method and the second was used to confirm the validity of the method as well as to compare the results of the detuned and the retuned cases.

TABLE I
Q COMPARISON BETWEEN THE BANDPASS METHOD AND THE SINGLE FREQUENCY METHOD (CIRCULAR CYLINDRICAL CAVITY)

F(GHz)	$\|\Gamma\|$	Q FACTOR			COUPLING CONDITION
		BANDPASS	SINGLE FREQ.	DIFFERENCE(%)	
2.9550	0.469	7129	7122	0.1	OVER COUPLED
2.9550	0.425	6408	6374	0.5	
2.9549	0.348	5414	5322	1.7	
2.9549	0.332	5247	5136	2.1	
2.9549	0.297	4880	4751	2.6	
2.9549	0.293	4829	4706	2.6	
2.9549	0.032	2850	2749	3.7	
2.9450	0.177	1768	1770	0.1	UNDER COUPLED
2.9450	0.190	1712	1772	0.6	
2.9450	0.227	1581	1594	0.8	
2.9449	0.352	1174	1213	3.3	
2.9449	0.414	997	1048	5.2	
2.9448	0.521	724	797	10.0	
2.9447	0.563	626	708	12.9	
2.9447	0.684	398	476	19.6	

B. Procedures

Prior to the measurements, the constants C_0 of the cavities were determined by the standard bandpass method in order to allow us to present absolute Q value data. In this way, the data can show not only the difference between the single-frequency method and the standard method but also indicate the absolute Q range covered.

1) Circularly Cylindrical Cavity: In order to verify (12), we wish to keep the frequency shift and the field disturbance as small as possible while the Q factor varies during the introduction of the loss into the cavity. To achieve this, about 0.13 g of dry salt, NaCl, was placed in the glass tube that had been inserted into the cavity and its resonant performance was recorded. Subsequently, a very small drop of water was added by letting it slide slowly along the inner surface of the tube wall as illustrated in Fig. 3(a). On absorbing the water, the loss of the salt increases dramatically but the dielectric constant of the mixture does not change much. This leads to a significant Q reduction with little variation in resonant frequency and field configurations. During this period, the Q measurements, using both the bandpass and the single-frequency methods, were performed automatically via the previously mentioned computer program. The results are listed in Table I for two different coupling conditions. It shows that the results for the single frequency Q measurement method compare very favorably with those of the bandpass method. It should be pointed out that the resonant data on which the table is based were acquired semicontinuously, one reading every 0.025 s., as the salt was absorbing the water. As expected, the percentage difference between the two methods increases as the disturbance, because of the dielectric sample, becomes larger.

2) Re-entrant Coaxial Cavity: With this cavity, the validity of (12) was again confirmed but, in addition, dissimilarities of the results under perturbation conditions after the manner of the detuning and the retuning were revealed. The retuning was provided by a metal tuning stub as shown in Fig. 3(b). We inserted a 0.1-N NaCl solution in a glass tube through the cavity hole into the high E-field region in the gap. Because of the high dielectric constant and the high loss

TABLE II
Q COMPARISON BETWEEN THE BANDPASS METHOD AND THE SINGLE FREQUENCY METHOD WITH FREE RESONANT FREQUENCY SHIFT I.E., THE DETUNING CASE (RE-ENTRANT CAVITY, UNDER COUPLED)

F(GHz)	$\|\Gamma\|$	Q FACTOR		
		BANDPASS	SINGLE FREQ.	DIFFERENCE(%)
2.4042	0.154	1082	1110	2.6
2.4028	0.199	981	1012	3.2
2.4012	0.243	908	922	1.5
2.4003	0.267	857	876	2.3
2.3998	0.282	818	848	3.7
2.3983	0.324	711	774	8.8
2.3969	0.361	632	712	12.6
2.3958	0.387	568	670	17.8
2.3934	0.451	462	573	24.0

TABLE III
Q COMPARISON BETWEEN THE BANDPASS METHOD AND THE SINGLE-FREQUENCY METHOD AT A FIXED RESONANT FREQUENCY, I E, THE RETUNING CASE (RE-ENTRANT CAVITY, UNDER COUPLED, f = 2 4046 GHz)

$\|\Gamma\|$	Q FACTOR		
	BANDPASS	SINGLE FREQ.	DIFFERENCE(%)
0.131	1129	1195	5.9
0.165	1032	1085	5.1
0.232	879	943	7.3
0.254	854	900	5.4
0.290	772	833	7.9
0.335	696	753	8.3
0.366	645	702	8.8
0.417	568	622	9.5
0.498	460	507	10.2
0.607	338	371	9.5

tangent of this sample, the Q factor as well as the resonant frequency were altered significantly as the sample was being inserted. During this insertion period, resonant data was again obtained on a semicontinuous basis. To obtain the data for the retuning case, we repeated the Q measurements and retuned the cavity after each change in disturbance to its original resonant frequency by adjusting the metal stub tuner. The Q, the resonant frequency, and the reflection coefficient for the detuning and the retuning cases are listed in Tables II and III, respectively for comparison.

C. Discussion

It is clear that the Q values from the single-frequency method agree well with those from the standard bandpass method over a wide range of Q values. For example, the difference is less than 3.7% for a Q range of 7000 to 3000 and less than 20% for 1800 to 400. The discrepancy from the standard bandpass method's data increases with the increase of the perturbation.

As more disturbance is introduced, the difference in the measured Q values increases rapidly in the detuned case whereas the difference remains relatively constant in the retuned case. This confirms the prediction made in the previous theoretical analysis that the Q measured in the retuned case will be more accurate than that measured in the detuned case. Retuning effectively increases the allowed perturbation range. This is believed to result from the rebalancing of the cavity energy by the retuning, which removes the effect of the energy change on the parameter S.

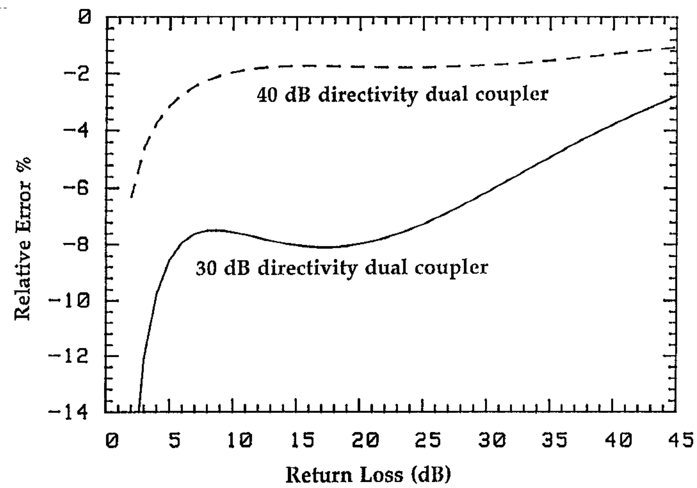

Fig. 4. Relative Q error vs. cavity return loss using a simple reflectometer.

IV. ERROR ANALYSIS OF RELATIVE Q MEASUREMENT

In the foregoing, the perturbation theory for the single-frequency Q measurement is proved to be valid theoretically and experimentally within a certain error. This error is analyzed more fully in this section. From the Q expression of (7), we can write the general form for the relative error in Q as

$$\frac{\Delta Q}{Q} = \frac{\Delta \omega}{\omega} + \frac{\Delta S}{S} + 2\frac{\Delta n}{n} - \frac{\Delta Z_0}{Z_0} - \frac{\Delta \overline{R}}{\overline{R}} \tag{22}$$

where $\Delta Z_0/Z_0$ can be considered to be zero. $\Delta S/S$, $\Delta n/n$ and $\Delta\omega/\omega$ are negligible, as suggested in the previous perturbation analysis. Therefore, when the relative variation of Q is of interest, as is true in many practical cases, the only major error source is the $\Delta\overline{R}/\overline{R}$ term. This resonant input resistance error is related to the reflection coefficient by

$$\frac{\Delta \overline{R}}{\overline{R}} = \frac{2\Delta|\Gamma|}{1-|\Gamma|} \tag{23}$$

Since

$$|\Gamma| = 10^{\frac{RL}{20}}$$

where RL is the return loss in dB, (23) becomes

$$\frac{\Delta \overline{R}}{\overline{R}} = \frac{|\Gamma|\ln 10}{10(1-|\Gamma|)}\Delta(RL) \tag{24}$$

For a dual directional coupler of 30 dB directivity, the lower and upper limits of RL are [5]

$$\Delta(RL)^{-} = 0.336 - 0.0025|RL|^{2\ 373} \tag{25}$$

and

$$\Delta(RL)^{+} = 0.354 + 0.0028|RL|^{2.223} \tag{26}$$

respectively. Based on (24)–(26) the relative error of $\overline{R}$ vs. RL is plotted in Fig. 4 for a 30-dB directivity coupler. For comparison, the curve for a 40 dB directivity dual directional coupler is presented in Fig. 4 also.

These curves demonstrate that the relative Q error is less than ±8% for a 30-dB directivity coupler and a minimum return loss of 5 dB. If a 40-dB directivity coupler is used, however, a substantial accuracy improvement is achieved, with the error remaining nearly constant over a large return loss range. This contrasts with the error behavior of Q for the typical bandpass method as revealed in a study done in [2], in which they show a minimum error of $\pm 10\%$ for the 30-dB directivity case over only a narrow return loss range. The error of the single-frequency Q measurement tends to decrease when the cavity input impedance approaches a matching condition. This is an obvious advantage for resonant systems operating near their matched condition. Furthermore, the present method eliminates the need for the resonant frequency f_0 and the 3-dB bandwidth Δf_0 measurements. Therefore, power and frequency variations would have no direct effect on the measured Q factor. Moreover, a single-frequency reflectometer can be tuned to give a much higher effective directivity at the fixed frequency, thereby further decreasing the error.

V. Application of the Relative Q Measurement

We will illustrate how the single-frequency relative Q measurement can be employed in the determination of the dielectric loss factor.

It is well known that the dielectric loss factor can be determined from the drop in Q factor resulting from the loss of the sample inserted in the cavity, as [2]

$$\varepsilon'' = \frac{\varepsilon'}{F}\left(\frac{1}{Q} - \frac{1}{Q_0}\right) \tag{27}$$

where Q_0 and Q is the cavity Q before and after the sample is loaded respectively, F is the filling factor, and ε' and ε'' are the dielectric constant and loss factor of the sample, respectively. Equation (27) can also be expressed as

$$Q_{0_{\varepsilon''}} = \frac{\varepsilon'}{F}\left(\frac{Q_0}{Q} - 1\right) \tag{28}$$

It is clear that the loss factor normalized with respect to Q_0 can be determined with the measurement of the relative Q, Q_0/Q. On the other hand, in fact, in most cases, especially in the complex sample geometry cases, the filling factor F is a calibrated factor. Therefore, if FQ_0, instead of F, is calibrated, then the absolute value of ε'' can be determined through Q_0/Q. Throughout the process, neither C_0 nor the absolute Q need to be determined.

VI. Conclusions

It has been proved that a Q factor can be measured at a single frequency condition. Moreover, perturbation theory has been applied to the measurement of the Q factor, proving that the Q factor is directly or inversely proportional to the input resonant resistance of a cavity when a moderate perturbation condition is satisfied. This principle has led to the development of the single-frequency relative Q measurement method. In addition, the perturbation condition of invariance of the field configuration is required strictly only for the usually small coupling region in the cavity, which is a less stringent condition than that required in the traditional perturbation approach. It is found that a retuned, perturbed resonator can yield a more accurate relative Q factor result. The experimental data from cylindrical and coaxial cavities confirms the validity of this method over a wide range of Q factors. An error analysis shows a more favorable performance compared to the bandpass method. As a relative method, the single-frequency method can be applied to arbitrary cavities satisfying the perturbation stipulation.

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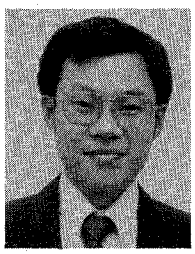

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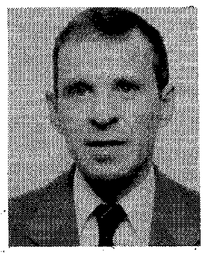

Wayne R. Tinga, received the Ph.D. degree in electrical engineering in 1969 from the University of Alberta, Edmonton, Alberta, Canada where he became assistant professor in 1970, associate professor in 1974, and full professor in 1980. He was on leave with Littton Microwave Cooking R/D Labs from 1979–1980, where he worked on developing one of his patents on frequency agile computer-controlled microwave ovens. He is a Fellow in the International Microwave Power Institute (IMPI) and served IMPI in many capacities since its inception in 1966 as President, Board Chairman, Executive Director, Symposium Chairman, and Technical Program Co-chairman. He also served on many technical committees dealing with Frequency Allocations, Radio Frequency Interference and Safety Standards. He served as Associate Editor of the Journal Microwave Power (1970–1978), its Editorial Review Board (1978–present), and was on the review board of IEEE Trans. Geoscience and Remote Sensing (1988–1991). From 1983–1986, he was on leave to Dordt College, Sioux City, Iowa, where he designed and implemented a 4-year electrical engineering program. After his return to the University of Alberta, he was acting Director of Computer Engineering (1987–1988). Presently he is Associate Chair of the Department of Electrical Engineering there.

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